PG – 141

III Semester M.Sc. Degree Examination, December 2014
(RNS) (Y2K11 Scheme)
MATHEMATICS
M 305 : Mathematical Methods

Time : 3 Hours Max. Marks : 80

Instructions : *1)* ***All*** *questions have* ***equal*** *marks.*
2) Answer ***any five*** *questions choosing atleast* ***two*** *from* ***each*** *Part.*

PART – A

1. a) Solve $\phi(x) = e^{-x} - 2\int_0^1 \cos(x-y)\,\phi(y)\,dy$ using the Laplace transform technique.

   b) Solve the integral equation $\phi'(t) = t + \int_0^t \cos x\,\phi(x)\,dx, \phi(0) = 2$ using the Laplace transform technique.

   c) Obtain the Fourier integral representation of a periodic function. Also, obtain the Fourier sine and cosine integral representations of the function. **(5+5+6)**

2. a) Find the Fourier cosine transform of $f(x) = \dfrac{1}{1+x^2}$.

   b) Derive the Mellin transform of the integrals and also find the Mellin transform of sinx and cosx.

   c) Determine the solution of integral equation $\psi(x) = x + \lambda\int_0^1 (xy^2 + x^2 y)\,\psi(y)\,dy$ using the separable kernal. **(5+6+5)**

3. a) Derive the Neumann series for a nonhomogeneous Fredholm integral equation of a second kind. Also discuss its convergence.

   b) Find the resolvant kernal of the Volterra integral equation : $k(x, y) = \dfrac{2+\cos x}{2+\cos y}$.

   c) Obtain an equivalent integral equation for an n^{th} order nonhomogeneous initial value problem. **(6+4+6)**

P.T.O.

4. a) Prove that the eigenvalues of symmetric kernal are real.

   b) State and prove the Hilbert-Scmidt theorem. **(8+8)**

PART – B

5. a) Obtain different expressions for the leading order asymptotic expansions of the method of stationary phase.

   b) Determine an asymptotic expansion of $I(x)=\int_0^\infty e^{-xt-1/t}\,dt$ as $x\to\infty$.

   c) Use Watson Lemma to find an asymptotic expansion of the integral

   $I(x)=\int_1^\infty (s^2-1)^{-1/2} e^{-xs}\,ds$ as $x\to\infty$. **(6+6+4)**

6. a) Find the asymptotic behavior of $I(x)=\int_0^1 e^{ixt^2}\,dt$ as $x\to\infty$. Using the method of steepest descent.

   b) Obtain the 3 - term perturbation series solution of the equation $f'''+ff''=0$.

   with the conditions $f(0)=0,\ f'(0)=1,\ f''(0)=0$. **(8+8)**

7. a) Determine the 2 - term uniformly valid solution of the problem :

   $\varepsilon y''+(1+x)y'+y=0$ with $y(0)=1, y(1)=1$.

   b) Find the periodic solution of the problem $y''+y+\varepsilon y^3=0,\ y(0)=a,\ y'(0)=0$ Using the Poincare – Lindstedt method. **(8+8)**

8. Show that the general nonlinear Riccati equation has a second order differential equation. Also solve the equation $f'''+ff''+f'^2=0$ with $f(0)=0,\ f'(0)=1,\ f''(0)=0$. **16**
